Review For Mat 191 Final Exam

Section 1

Perform the indicated operation(s) and simplify. Rationalize denominators when necessary, and write all complex numbers in the form \((a + bi)\).

1. \((6 + 5i) - (2 - 3i)\)  
   \[\text{Solution: } 4 + 8i\]

2. \((2 - i)(4 + 3i)\)  
   \[\text{Solution: } 11 + 2i\]

3. \(\frac{2 + 3i}{4 - 2i}\)  
   \[\text{Solution: } \frac{1}{10} + \frac{3}{2}i\]

4. \(\frac{4}{4 - 5i}\)  
   \[\text{Solution: } \frac{16}{41} + \frac{20}{41}i\]

5. \(i^{45}\)  
   \[\text{Solution: } -1\]

6. \(\left(2 + \sqrt{-2}\right)\left(2 - \sqrt{-2}\right)\)  
   \[\text{Solution: } 6\]

7. \(\left(3 - \sqrt{-7}\right)^2\)  
   \[\text{Solution: } 2 - 6i\sqrt{7}\]

8. \(\sqrt{6x^4y} - 3x\sqrt{2}xy\)  
   \[\text{Solution: } -3x\sqrt{2xy}\]

9. \(\sqrt{\frac{32x^2}{9x}}\)  
   \[\text{Solution: } \sqrt{\frac{2x}{1-x}}\]

10. \(\frac{2x^2 + 6x}{x^2 + 1}\)  
    \[\text{Solution: } 2\]

11. \(\left(\frac{8x^6y^7}{x^2y^2}\right)^{\frac{1}{3}}\)  
    \[\text{Solution: } 2xy^3\]

Section 2

Solve the following for \(x\):

1. \(\frac{2}{x-2} - \frac{3}{x+3} + \frac{10}{(x+5)(x-2)}\)
   \[\text{Solution: } \{6\}\]

2. \(\sqrt{3x + 4} + 2 = 0\)
   \[\text{Solution: No solution}\]

3. \(\sqrt{15} - 2x = x\)
   \[\text{Solution: } \{3\}\]

4. \(2x^2 + 5x + 12 = 0\)
   \[\text{Solution: } \left\{\frac{-5 \pm \sqrt{2}}{3}\right\}\]

5. \((5x^2 - 6)^{\frac{1}{2}} = x\)
   \[\text{Solution: } \left\{\sqrt{3}, \sqrt{2}\right\}\]

6. \(3x^2 + 6x = -1\)
   \[\text{Solution: } (-\infty, \infty)\]

7. \(3x - 11 < 4\) or \(3x - 11 \geq 8\)
   \[\text{Solution: } (-\infty, 4) \cup [5, \infty)\]

8. \(2x - 5 \geq 3\) and \(3x - 11 > 1\)
   \[\text{Solution: } [-1, \frac{10}{3}]\]

9. \(x^2 + x > 12\)
   \[\text{Solution: } (-\infty, -4) \cup (3, \infty)\]

10. \(\frac{(x + 3)(x - 4)}{x - 2} \leq 0\)
    \[\text{Solution: } (-\infty, -3) \cup (2, 4)\]

11. \(|2 - 3x| - 5 = 12\)
    \[\text{Solution: } (-\infty, 7) \cup [9, \infty)\]

12. \(|x - 8| \geq 1\)
    \[\text{Solution: } (-\infty, 7] \cup [9, \infty)\]

13. \(|3x + 5| > -9\)
    \[\text{Solution: } (-\infty, \infty)\]

14. \(|2x - 8| = 6\)
    \[\text{Solution: } \{4\}\]
15. $|5x - 5| < -4$  
\[ \text{Solution: No solution} \]

16. How many ounces of pure water should be added to 30 ounces of a 40% solution of hydrochloric acid to obtain a 30% solution of hydrochloric acid?  
\[ \text{Solution: 10 oz} \]

17. John has taken up a loan of $10,000, part at the rate of 8% per year and the rest at the rate of 18% per year. If the interest received totaled $1000, how much was loaned at 8%?  
\[ \text{Solution: } $8,000 \]

18. Two sisters Meg and Pat clean homes. Meg can finish a job in 8 hours, and Pat can do the same job in 7 hours. How long will it take them working together to complete the job?  
\[ \text{Solution: approx 4 hours} \]

19. The length of a rectangle is 2 inches more than its width. If the diagonal of the rectangle is 10 inches long, find the length and width of the rectangle.  
\[ l = 8, \quad w = 6 \]

20. How long will it take for $2500 to triple if it is invested in a savings account that pays 4.5% interest compounded continuously?  
\[ \text{Solution: approx 24 years} \]

21. Suppose $3000 is invested into an account paying 6% interest compounded quarterly. Find the balance in the account after 5 years.  
\[ \text{Solution: } $4040.56 \]

22. The perimeter of a rectangular area is 220 meters. If l is the length of one side, find the maximum area that can be enclosed.  
\[ \text{Solution: } 3025 \, m^2 \]

24. Two cars leave a city at the same time traveling in opposite directions. At the end of 7 hours they are 910 miles apart. What is the speed of each car, if one traveled 10 mph. faster than the other?  
\[ \text{Solution: 60 mph. and 70 mph.} \]

25. Find two integers with product of 1500 and sum of 80.  
\[ \text{Solution: 30, 50} \]

**Section 3**

1. Find the equation of the line that passes through the point $(-2, -3)$ and is perpendicular to $2x + 5y = 10$.  
\[ \text{Solution: } \frac{2}{5}x + 2 \]

2. Find the equation of the line with $x$-intercept $-2$ and parallel to the line $2x + 3y = 6$.  
\[ \text{Solution: } y = -\frac{2}{3}x - \frac{4}{3} \]

3. Find the distance between the points $(1, 3)$ and $(-4, 3)$. Also, find the midpoint of the line segment between the two points.  
\[ \text{Solution: 5, } (-\frac{1}{2}, 3) \]

4. Find the standard form of the equation of circle that has center $C(-2, 5)$ and radius $\sqrt{3}$.  
\[ \text{Solution: } (x + 2)^2 + (y - 5)^2 = 3 \]

5. Find the center and radius of $x^2 + y^2 - x + 2y + 1 = 0$  
\[ \text{Solution: Center } \left( \frac{1}{2}, -1 \right) \text{ Radius } \frac{1}{2} \]
6. Determine whether the graph of the following equations are symmetric with respect to either x-axis or y-axis. 
   a. \(|x| = |y| - 2\)  
   b. \(x^2 = y - 3\)  
   Solution: a. Symmetric with respect to x-axis and y-axis  
              b. Not symmetric

7. Determine whether the given functions are even, odd or neither. 
   a. \(f(x) = |x| + 2\)  
   b. \(g(x) = \frac{x}{x^3 - 1}\)  
   Solution: a. Even  
              b. Neither

8. Suppose the manufacturer of a washing machine has found that when the unit price is \(p\) dollars, the revenue, \(R\), in dollars is \(R = -5p^2 + 5000p\). What unit price must be established for the washing machines to maximize the revenue? What is the revenue?  
   Solution: Price $500  
              Revenue $1,250,000

9. A store owner buys a new printer for $750. It is estimated that after 10 years the value of printer will be $100. Find the linear function demonstrating this relationship, and use that to determine the value of printer after 5 years.  
   Solution: \(Y = -65t + 750\); after 5 years value of printer is $425.  

Graph using the graphing techniques.

10. \(y = -(x - 2)^2 + 2\)

11. \(y = (x - 3)^3 + 2\)
12. \( y = \sqrt{x-1} + 2 \)

![Graph of \( y = \sqrt{x-1} + 2 \)]

13. \( y = -\frac{1}{3}|x+1| \)

![Graph of \( y = -\frac{1}{3}|x+1| \)]

Find the domain of each function.

14. \( f(x) = \frac{2x}{5x-10} \)
   Solution: \( \{x \mid x \neq 2\} \)

15. \( f(x) = 3x^3 + 5x^2 - 1 \)
   Solution: \( (-\infty, \infty) \)

16. \( f(x) = \sqrt{x^2} - 5x \)
   Solution: \( (-\infty, -3] \cup [3, \infty) \)

17. \( f(x) = \frac{3x}{3x^2 - 7x - 6} \)
   Solution: \( \{x \mid x \neq -\frac{2}{3}, x \neq 3\} \)

Identify the equation that defines \( y \) as a function of \( x \).

18. \( 3x - 5y = 2 \)

19. \( x^2 + y^2 = 8 \)

20. \( x^2 - y = 4 \)

21. \( y^2 - 3x = 5 \)

Evaluate the indicated functions, where \( f(x) = x^2 - 2x + 1 \) and \( g(x) = x - 3 \).

22. \( (f+g)(2) \)
   Solution: 0

23. \( \left( \frac{f}{g} \right)(-4) \)
   Solution: \(-\frac{25}{7}\)

Find \( f \circ g \) for the given functions \( f \) and \( g \).

24. \( f(x) = \sqrt{x+4}, g(x) = \frac{1}{x} \)
   Solution: \( f \circ g = \frac{\sqrt{x+4x^2}}{x} \)

25. \( f(x) = -x^2 - 7, g(x) = x + 1 \)
   Solution: \( f \circ g = -(x^3 - 3x^2 - 3x - 8) \)

Scientific Notebook: On Line Mathematics
26. The maximum safe load for a horizontal beam varies jointly with the width of the beam and the square of the thickness of the beam and inversely with its length. If a 96 inch beam will support up to 700 pounds when the beam is 4 inches wide and 2 inches thick, what is the maximum safe load in a similar beam 144 inches long, 8 inches wide, and 2 inches thick?

Solution: 933.33 lb.

Section 4

Solve for x.

1. \(2^x = 47\) \hspace{2cm} \text{Solution:} x \approx 3.55

2. \(\ln x - \ln(x - 1) = 1\) \hspace{2cm} \text{Solution:} x \approx 1.58

3. \(e^{2x} = 4\) \hspace{2cm} \text{Solution:} x \approx 0.69

4. \(e^{\ln(5x^2)} = 7\) \hspace{2cm} \text{Solution:} x = 1

5. \(\log_4 x - \log_4(x - 1) = \frac{1}{2}\) \hspace{2cm} \text{Solution:} x = 2

Write the expression as a logarithm of a single quantity.

6. \(\ln x - 2[\ln(x + 2) + \ln(x - 2)]\) \hspace{2cm} \text{Solution:} \frac{x}{(x^2 - 4)^2}

7. \(\log x - 3 \log(x - 1)\) \hspace{2cm} \text{Solution:} \frac{x}{(x - 1)^3}

Write the expression as a sum, difference, and/or constant multiple of logarithms.

8. \(\ln \frac{x}{\sqrt{x^2 + 1}}\) \hspace{2cm} \text{Solution:} \ln x - \frac{1}{2} \ln(x^2 + 1)

9. \(\log \frac{x^4 \sqrt{y}}{z^5}\) \hspace{2cm} \text{Solution:} 4 \log x + \frac{1}{2} \log y - 5 \log z

10. Find domain of \(f(x) = \ln(2 - x)\). \hspace{2cm} \text{Solution:} (-\infty, 2)

Use graphing techniques to sketch the graph of each function. Also, state the domain and range of \(f\).

11. \(f(x) = 2^{x+1} + 2\)

\[\text{Domain: } (-\infty, \infty) \hspace{1cm} \text{Range: } (2, \infty)\]
12. \( f(x) = -\log(x + 2) \)

\[
\begin{array}{c}
-4 & -2 & 0 & 2 & 4 \\
-4 & -2 & 2 & 4 & 6
\end{array}
\]

Domain: \((-2, \infty)\)  Range: \((-\infty, \infty)\)

**Section 5**

Solve the following systems by any method.

1. \[
\begin{aligned}
3x - 2y &= 7 \\
6x - 4y &= 4
\end{aligned}
\]
   Solution: No solution

2. \[
\begin{aligned}
2x - 3y &= -12 \\
x + 2y &= 8
\end{aligned}
\]
   Solution: \((0,4)\)

3. \[
\begin{aligned}
2x - 4 &= 3y \\
6x &= 9y + 12
\end{aligned}
\]
   Solution: \(\{(x,y) \mid 2x - 3y = 4\}\)

4. \[
\begin{aligned}
x + y &= 1 \\
y - z &= 1 \\
x + y + z &= 2
\end{aligned}
\]
   Solution: \((-1,2,1)\)

5. \[
\begin{aligned}
4x + y + 2z &= -2 \\
x - y - z &= -9 \\
-5x - 5y - 7z &= 100
\end{aligned}
\]
   Solution: No solution
THINGS TO MEMORIZE FOR THE FINAL EXAM

Unit 1  

Rules for Exponents

The product rule: If we have the same base, and are multiplying, we add the exponents. Ex., \(x^m x^n = x^{m+n}\)

The quotient rule: If we have the same base, and are dividing, we subtract the exponents. Ex., \(x^m / x^n = x^{m-n}\)

The power rule: If we raise a power to a power, then we multiply the exponents. Ex., \((x^m)^n = x^{mn}\)

The definition of negative exponents: If a negative exponent appears in the denominator of a fraction, we move it to the numerator and make the exponent positive. If a negative exponent appears in the numerator of a fraction, we move it to the denominator and make it positive.

Powers of \(i\):

\[
\begin{align*}
 i^0 &= 1 \\
 i &= \sqrt{-1} \\
 i^2 &= -1 \\
 i^3 &= i\times i^2 = i \times -1 = -i \\
 i^4 &= i\times i^3 = i \times -i = -i^2 = -(-1) = 1
\end{align*}
\]
The Equations

A linear equation: a first degree equation, meaning its highest exponent is one and it contains no variables in a denominator. Also, in order to be an equation, it must have an equal sign.

A quadratic equation: a second degree equation, meaning its highest exponent is two and it contains no variables in a denominator. Also, in order to be an equation, it must have an equal sign. Before we start to solve a quadratic equation, we must always make sure that it is in standard form, which means the x squared term comes first, the x term second and the constant term third.

A rational equation: an equation which contains a rational expression (a fraction that contains polynomials in both the numerator and the denominator).

An equation which is quadratic in nature: To determine whether an equation is quadratic in nature, first put the equation in descending order. If the middle term without coefficient squared is the first term without coefficient, then the equation is quadratic in nature.

3rd and 4th degree equations: A third degree equation is an equation with a polynomial of highest exponent 3, such as \( x^3 - 27 = 0 \). A fourth degree equation is an equation with polynomial of highest degree 4, such as \( x^4 - 3x^3 - 10 = 0 \). When dealing with 3rd and 4th degree equations, first check that they are not quadratic in nature. If an equation is quadratic in nature, use U substitution to solve it.

Rational exponent equations: A rational exponent equation is an equation that contains rational exponents, such as \((2x - 5)^{3/2} = 8\). If an equation is of the form "something" to a rational exponent = "something else," we will consider it to be a rational exponent equation. If an equation contains more than one term to an exponent, it is most likely an equation which is quadratic in form, for example, \(3x^{2/3} - x^{1/3} + 6 = 0\). It is important to recognize the difference between the two.
absolute value inequality, we must break up the absolute value. How we break up the absolute value depends on whether we have a less than or a greater than symbol. Before we break up an absolute value, we must have the absolute value on one side by itself! If we have a “less than” symbol, we will put what is inside the absolute value in the middle as follows: \( |x| < 3 \), \(-3 < x < 3\). If we have a “greater than” symbol, we break the inequality up into two inequalities. Be sure to “flip” the inequality for the negative part. \( |x| > 3 \), \( x > 3 \) or \( x < -3 \).

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**The distance formula:** \( d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)} \)

**The midpoint formula:** \( (\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}) \)

**The Formula for slope:** \( m = \frac{(y_2 - y_1)}{(x_2 - x_1)} \)

**The point-slope equation of a line:** \( y - y_1 = m(x - x_1) \)

**The slope-intercept equation of a line:** \( y = mx + b \)

**The standard equation of a circle:**

Given the center of a circle, the point \((h,k)\) and the radius of the circle, \( r \), the standard equation of the circle will be \((x - h)^2 + (y - k)^2 = r^2\).

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**The rules of translation and reflection:**

Given the graph of \( f(x) \), to get the graph of \(-f(x)\), we reflect over the x-axis.

Given the graph of \( f(x) \), to get the graph of \( f(-x) \), we reflect over the y-axis.

Given the graph of \( f(x) \), to get the graph of \( f(x + c) \), move the graph left \( c \) units.

Given the graph of \( f(x) \), to get the graph of \( f(x - c) \), move the graph right \( c \) units.

Given the graph of \( f(x) \), to get the graph of \( f(x) + c \), move the graph up \( c \) units.

Given the graph of \( f(x) \), to get the graph of \( f(x) - c \), move the graph down \( c \) units.
because we solve them differently.

Radical equations: A radical equation is an equation which contains a radical.

Absolute value equations: An absolute value equation is an equation which contains absolute value symbols. The key to solving absolute value equations is to remember that they will have two solutions. For example, $|x| = 7$. Then $x = 7$ or $x = -7$; both have absolute value 7. Also remember that an absolute value is always positive or zero.

Inequalities

Linear inequalities: A linear inequality is the same as a linear equation, except it has an inequality symbol instead of an $=$ symbol. To solve a linear inequality, we will follow the same steps as when we solve a linear equation; we want to get $x$ by itself on one side. Be sure to put your answers in interval notation.

Non-linear inequalities: We learned about two types of non-linear inequalities: quadratic inequalities and rational inequalities. A quadratic inequality, like a quadratic equation, has highest exponent two. For example, $x^2 - 7x - 30 < 0$. A rational inequality, like a rational equation, has a fraction whose numerator and denominator are both polynomials. For example,

$$\frac{x-3}{x+2} > 0$$

We solve both by using the critical method.

Absolute value inequalities: An absolute value inequality is the same as an absolute value equation, except it has an inequality symbol instead of an $=$ symbol. To solve an